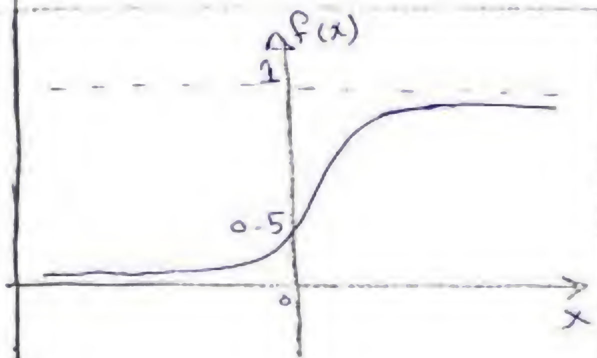
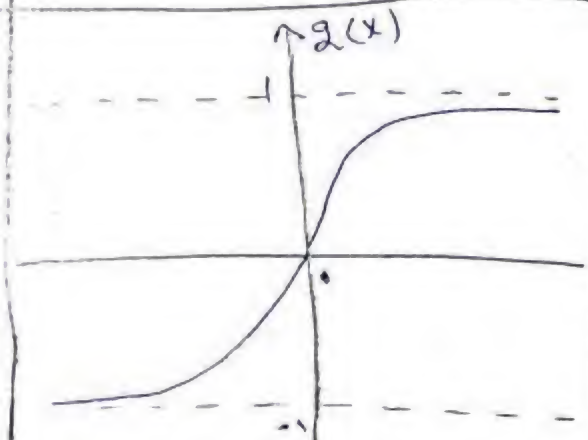
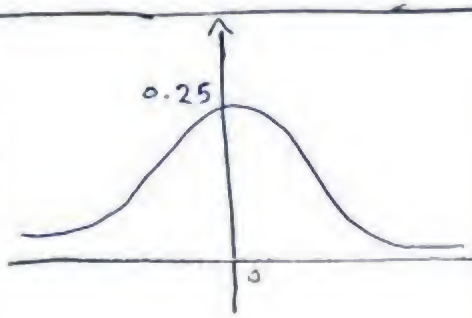
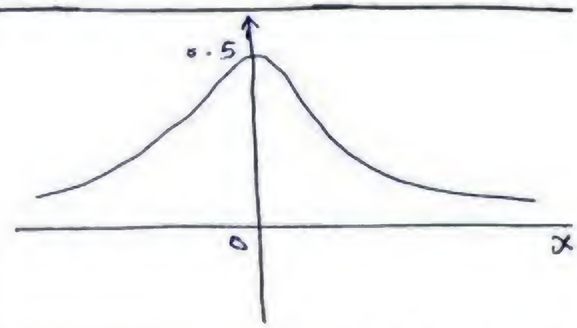
Lec 9 · Neural network

→ A comparison between binary and bipolar sigmoidal functions.

	Binary sigmoid	Bipolar sigmoid
Definition	$f(x) = \dfrac{1}{1+e^{-x}}$	$g(x) = \dfrac{2}{1+e^{-x}} - 1 = \dfrac{1-e^{-x}}{1+e^{-x}}$
Graph		
Range	$0 < f(x) < 1$	$-1 < g(x) < 1$
Relation between f(x), g(x)	$g(x) = 2f(x) - 1$	
Value of x	$x = \ln \dfrac{f(x)}{1-f(x)}$	$x = \ln\left[\dfrac{1+g(x)}{1-g(x)}\right]$

باقي المقارنة

Derivative	$\frac{df(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2}$ $= f(x)\left[1 - f(x)\right]$	$\frac{dg(x)}{dx} = \frac{2e^{-x}}{(1+e^{-x})^2}$ $= 0.5\left(1 - g^2(x)\right)$
Graph of derivative		
Range of values of derivative	$0 < \frac{df(x)}{dx} \leqslant 0.25$	$0 < \frac{dg(x)}{dx} \leqslant 0.5$
Relation between $\frac{dg(x)}{dx}$, $\frac{df(x)}{dx}$	$\frac{dg(x)}{dx} = 2\,\frac{df(x)}{dx}$	

Sigmoidal functions of the forms:

① $$f(x) = \frac{1}{1 + e^{-\alpha x}}$$

② $$g(x) = \frac{2}{1 + e^{-\alpha x}} - 1 = \frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}}$$

③ $$h(x) = \tanh(\alpha x)$$

where α is +ve Parameter

---

a) $$f(x) = \frac{1}{1 + e^{-\alpha x}}$$

verify that $$x = \frac{1}{\alpha} \ln \frac{f(x)}{1 - f(x)}$$

القانون ده موجود فيا قبل لكنه α كانت تساوى 1 .

Proof

$$f(x) = \frac{1}{1 + e^{-\alpha x}}$$

$$f(x) + e^{-\alpha x} f(x) = 1$$

ناخذ Ln للطرفين

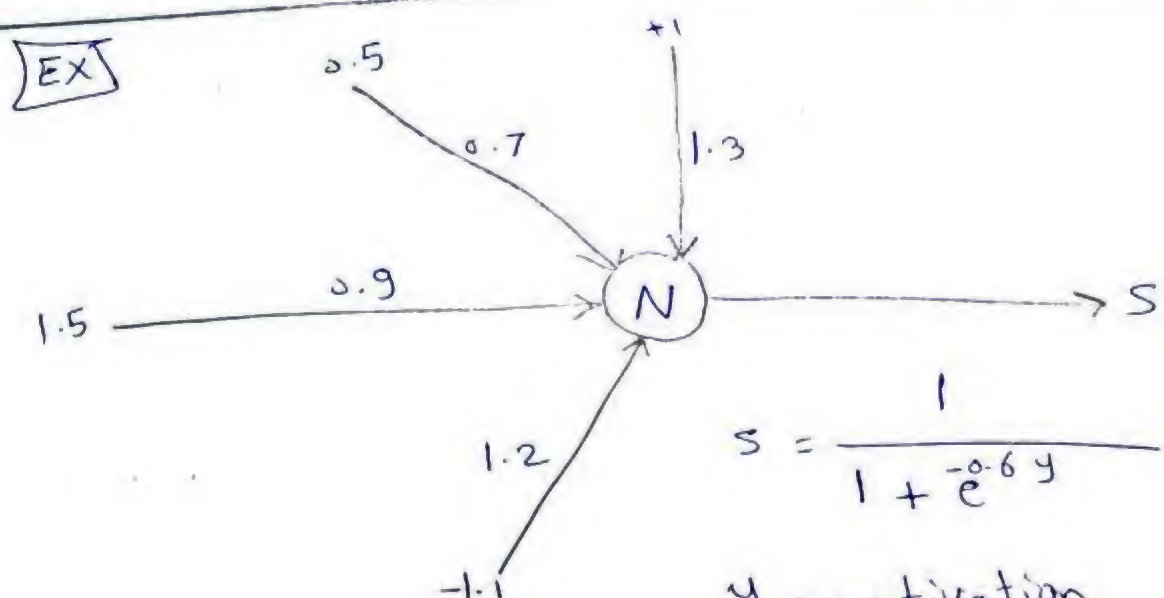
$$e^{-\alpha x} = \frac{1 - f(x)}{f(x)}$$

$$-\alpha x = Ln \frac{1 - f(x)}{f(x)}$$

$$\alpha x = Ln \frac{f(x)}{1 - f(x)} \Rightarrow x = \frac{1}{\alpha} Ln \frac{f(x)}{1 - f(x)}$$

$$\alpha = \frac{1}{x} Ln \frac{f(x)}{1 - f(x)} \quad \#$$

$\boxed{EX}$



$$S = \frac{1}{1 + e^{-0.6y}}$$

y = activation

$$y = (0.5)(0.7) + (1.5)(0.9) + (-1.1)(1.2) + (1.3)(1) = 1.68$$

- يستقبل نيورون مدخلات 0.6 ، 1.7 ، -1.5 بـ weights

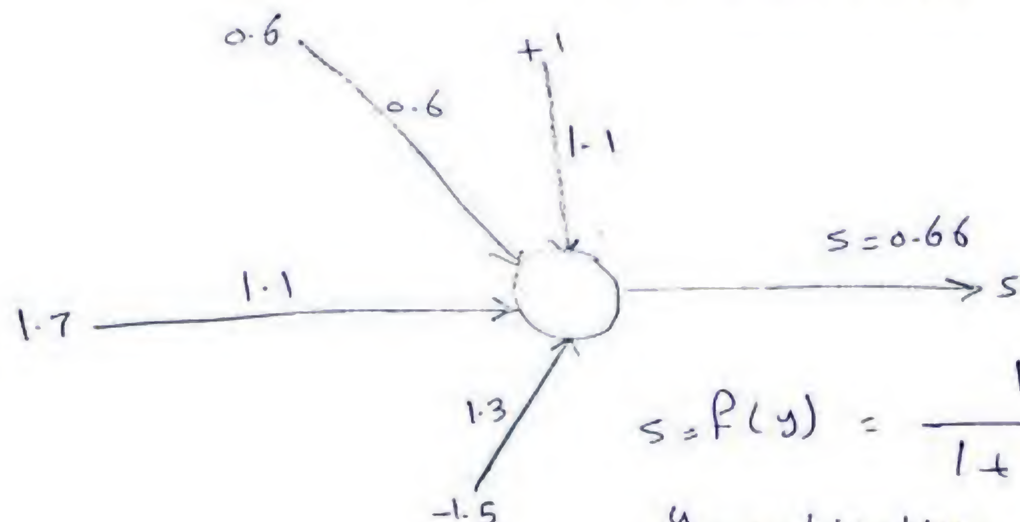
0.6 ، 1.1 ، 1.3 على الترتيب ، ويستخدم (sigmoidal fn.)

على الصورة حيث $f(y) = \dfrac{1}{1+e^{-\alpha y}}$

$y \longrightarrow$ activation ، $\alpha \longrightarrow$ +ve Parameter

- أوجد قيمة α بحيث إشارة الخرج تصبح 0.66 ،

اعتبر bias weight = 1.1 .



$$s = f(y) = \frac{1}{1+e^{-\alpha y}}$$

y = activation

Activation .

$$y = (0.6)(0.6) + (1.7)(1.1) + (-1.5)(1.3) + (1.1) = 1.38$$

$$\alpha = \frac{1}{y} \mathcal{L}n\left[\frac{f(y)}{1-f(y)}\right] = \frac{1}{y} \mathcal{L}n\left[\frac{s}{1-s}\right]$$

$$= \frac{1}{1.38} \mathcal{L}n\left(\frac{0.66}{1-0.66}\right) = 0.481$$

$$f(x) = \frac{1}{1 + e^{-\alpha x}}$$

verify that

$$\frac{df(x)}{d(x)} = \frac{\alpha\, e^{-\alpha x}}{(1 + e^{-\alpha x})^2}$$

$$= \alpha\, f(x)\, [1 - f(x)]$$

Proof $\quad f(x) = \frac{1}{1 + e^{-\alpha x}}$

$$\frac{df(x)}{dx} = \frac{\alpha\, e^{-\alpha x}}{(1 + e^{-\alpha x})^2}$$

$$= \frac{\alpha}{(1 + e^{-\alpha x})} \cdot \frac{e^{-\alpha x}}{(1 + e^{-\alpha x})}$$

$$= \frac{\alpha}{1 + e^{-\alpha x}} \cdot \frac{1 + e^{-\alpha x} - 1}{1 + e^{-\alpha x}}$$

$$= \frac{\alpha}{1 + e^{-\alpha x}} \left[1 - \frac{e^{-\alpha x}}{1 + e^{-\alpha x}} \right]$$

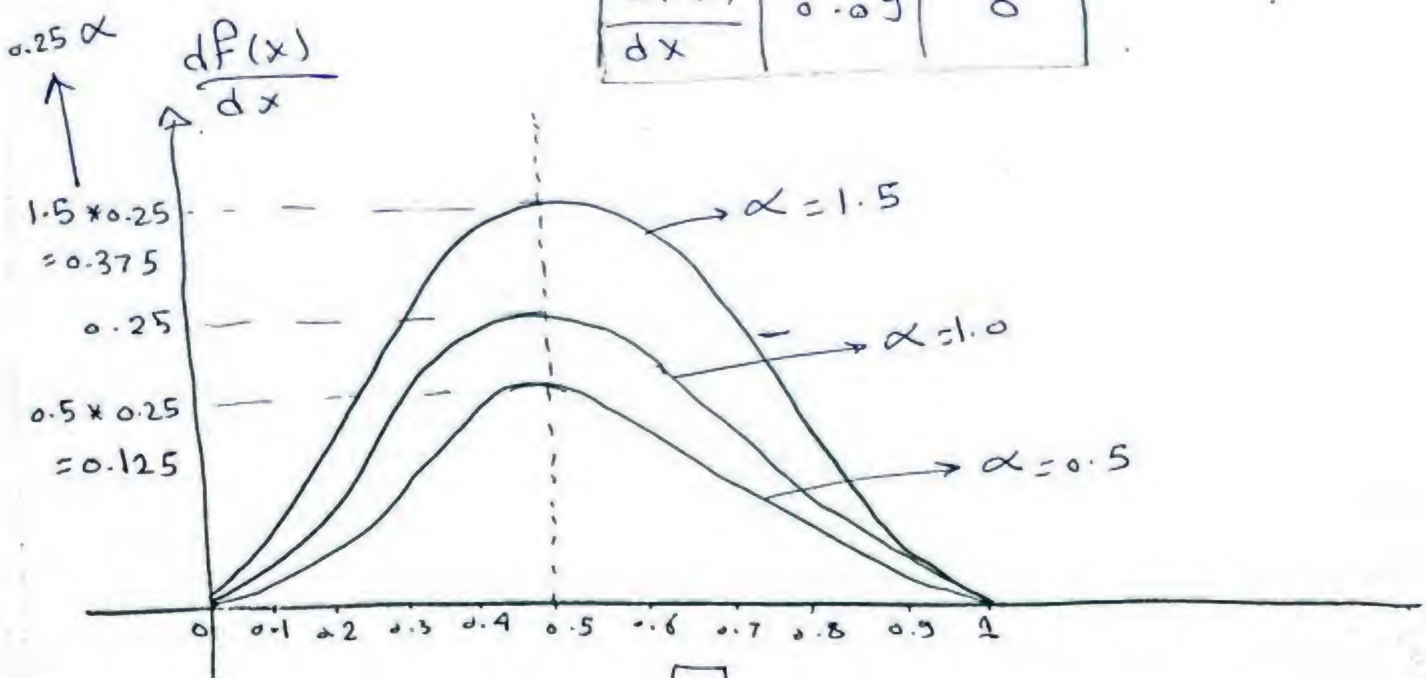
$$\frac{df(x)}{dx} = \alpha \, f(x)\left[1 - f(x)\right] \quad \#$$

$$\frac{df(x)}{dx} \quad \text{vs.} \quad f(x) \quad \text{"For } \alpha = 0.5, 1, 1.5$$

لو اثبت أن القيمة العظمى للتفاضل = α 0.25 , تحدث عندما $f(x) = 0.5$.

$$\frac{df(x)}{dx} = \alpha \, f(x)\left[1 - f(x)\right]$$

$f(x)$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\frac{df(x)}{dx}$	0	0.09				0.25				0.09	0



- يتضح من الشكل السابق أن القيمة العظمى للمشتقة $\frac{df(x)}{dx}$ هي $(0.25\,\alpha)$ وتكون عندما $f(x) = 0.5$.

ـ يمكن اثبات هذه العلاقة رياضياً عن طريق الآتي.
نجيب النهاية العظمى رياضياً بأن نضع $\frac{df(x)}{dx} = 0$

$$\frac{df(x)}{dx} = \alpha\, f(x)\,[1 - f(x)]$$

$$= \alpha\, f(x) - \alpha\, f^2(x)$$

$$\frac{d\left(\frac{df(x)}{dx}\right)}{d f(x)} = \alpha - 2\alpha f(x) = 0$$

$$f(x) = 0.5$$

$$\left.\frac{df(x)}{dx}\right|_{Max.} = \alpha * 0.5 \cdot [1 - 0.5]$$

$$= 0.25\,\alpha$$

— نفس الكلام على ال Bipolar.

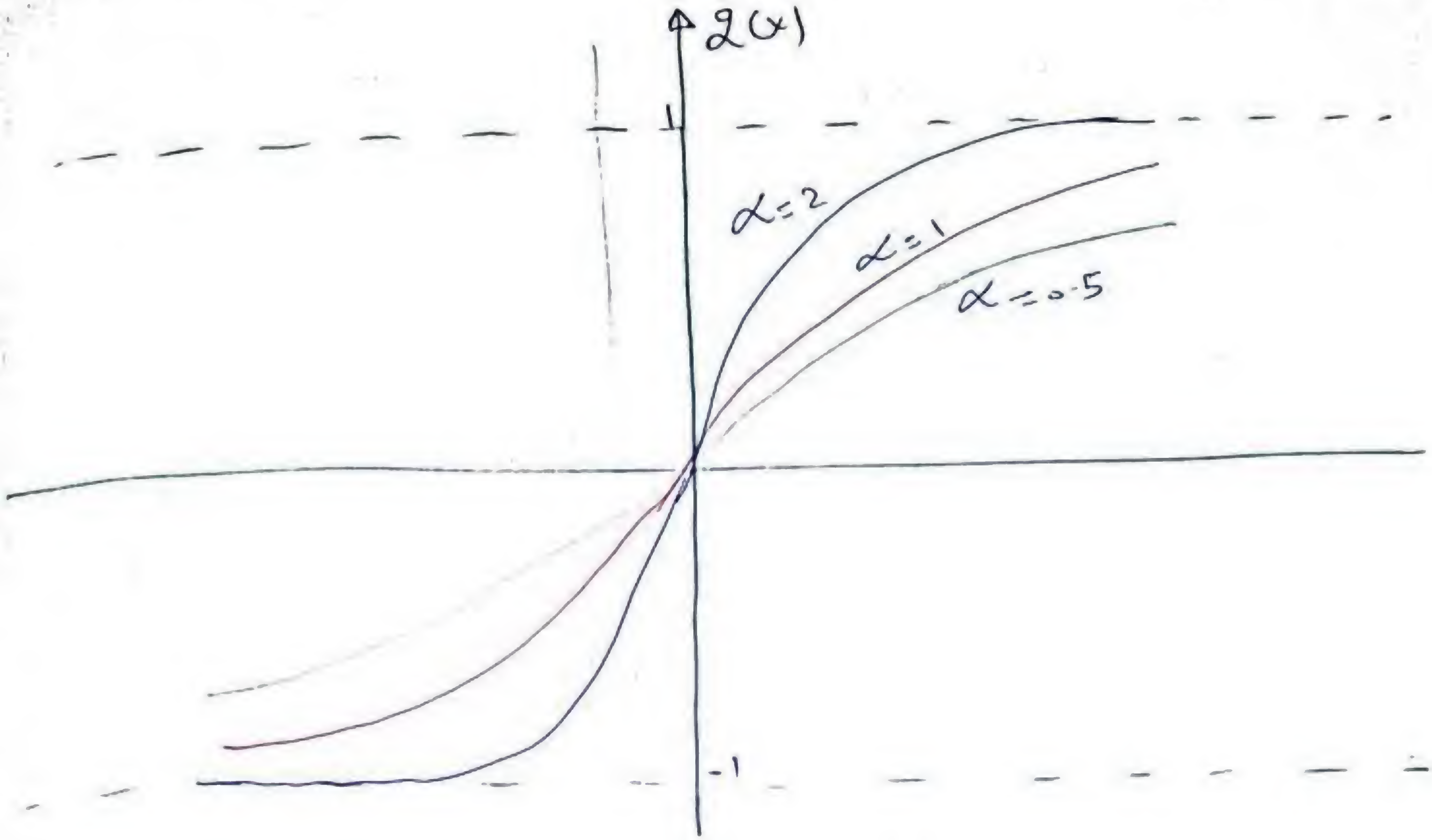
$$g(x) = \frac{2}{1 - e^{-\alpha x}} - 1 = \frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}}$$

Draw, on the same coordinate axes, the graphs of $g(x)$ fn. $\alpha = 0.5, 1, 2$ and comment on these graphs.

verify that $x = \frac{1}{\alpha} \ln\left[\frac{1 + g(x)}{1 - g(x)}\right]$

and, $\frac{dg(x)}{dx} = 0.5\,\alpha\left[1 - g^2(x)\right]$

x	$g(x), \alpha = 0.5$	$g(x), \alpha = 1$	$g(x), \alpha = 2$
-5	-0.848	-0.987	-0.999
-4	-0.762	-0.964	-0.998
⋮			
0	0	0	0
⋮			
4	0.762	0.964	0.998
5	0.848	0.987	0.999



ـــ مع زيادة قيم α فإن الشكل البياني يقترب أكثر وأكثر من المحور الرأسي ، وفي النهاية عندما $(\alpha \to \infty)$ فإن الدالة تصبح (Bipolar threshold function)

$$g(x) = \frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}}$$

$$g(x) + e^{-\alpha x} g(x) = 1 - e^{-\alpha x}$$

$$e^{-\alpha x} = \frac{1 - g(x)}{1 + g(x)}$$

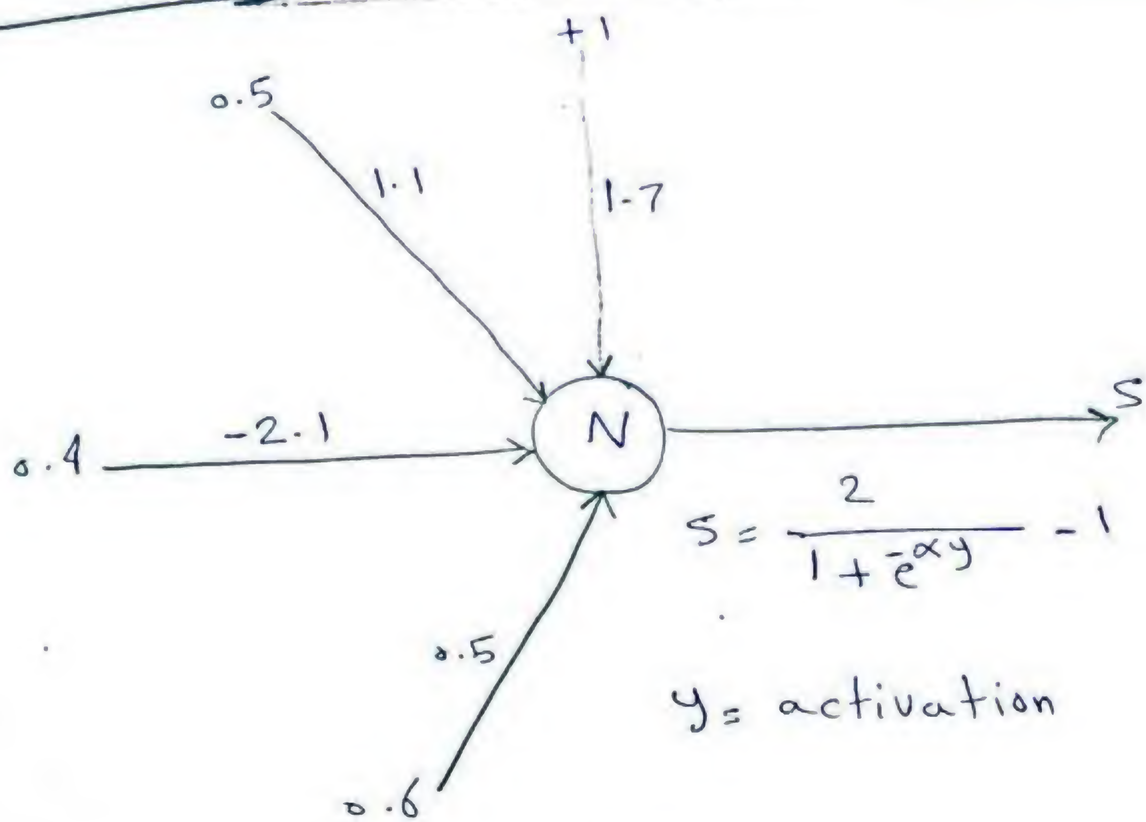
$$-\alpha x = \ln \frac{1 - g(x)}{1 + g(x)}$$

$$\boxed{x = \frac{1}{\alpha} \ln \left[\frac{1 + g(x)}{1 - g(x)} \right]}$$

$$g(x) = \frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}}$$

$$\frac{d\,g(x)}{dx} = \frac{(1+e^{-\alpha x})(\alpha e^{-\alpha x}) - (1 - e^{-\alpha x})(-\alpha e^{-\alpha x})}{(1 + e^{-\alpha x})^2}$$

$$= 0.5\,\alpha \left[1 - \left(\frac{1 - e^{-\alpha x}}{1 + e^{-\alpha x}} \right)^2 \right] = 0.5\,\alpha \left[1 - g^2(x) \right]$$



$$S = \frac{2}{1 + e^{-\alpha y}} - 1$$

y = activation

find the value of the Parameter α such that $S = 0.75$

Activation,

$$y = (0.5)(1.1) + (0.4)(-2.1)(0.6)(0.5) + 1.7 = 1.71$$

$$y = \frac{1}{\alpha} \ln\left(\frac{1+S}{1-S}\right) \qquad S = f(y)$$

$$\alpha = \frac{1}{y} \ln\left(\frac{1+S}{1-S}\right)$$

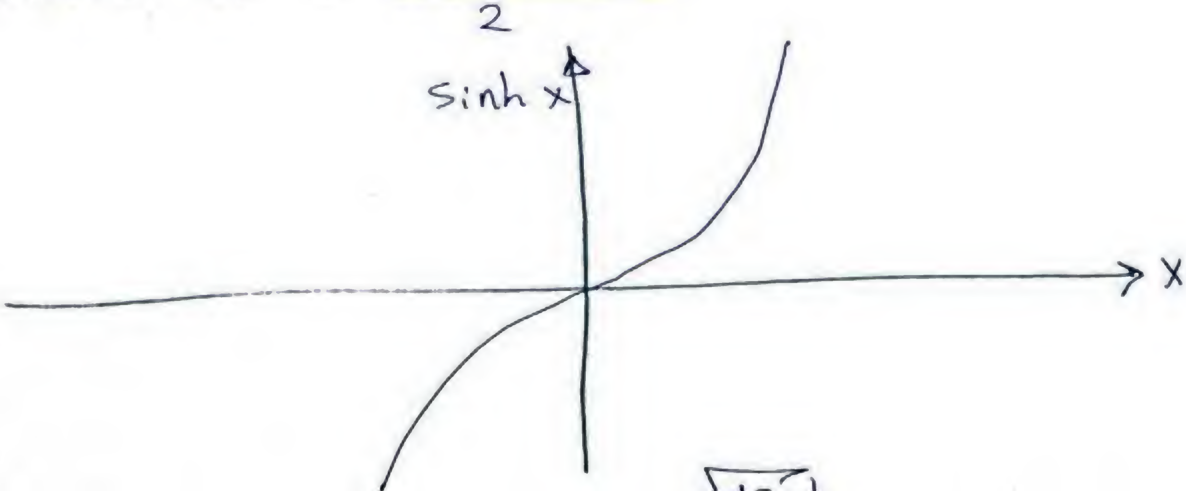
$\underline{S = 0.75}$

$$\alpha = \frac{1}{1.71} \ln\left(\frac{1+0.75}{1-0.75}\right) = 1.138$$

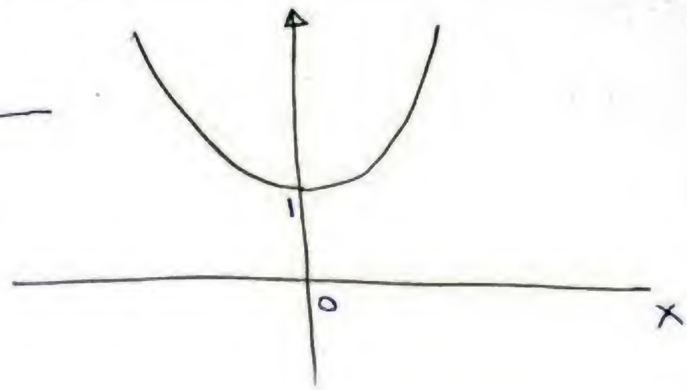
---

* Hyperbolic functions

$$\sinh x = \frac{e^{x} - e^{-x}}{2}$$

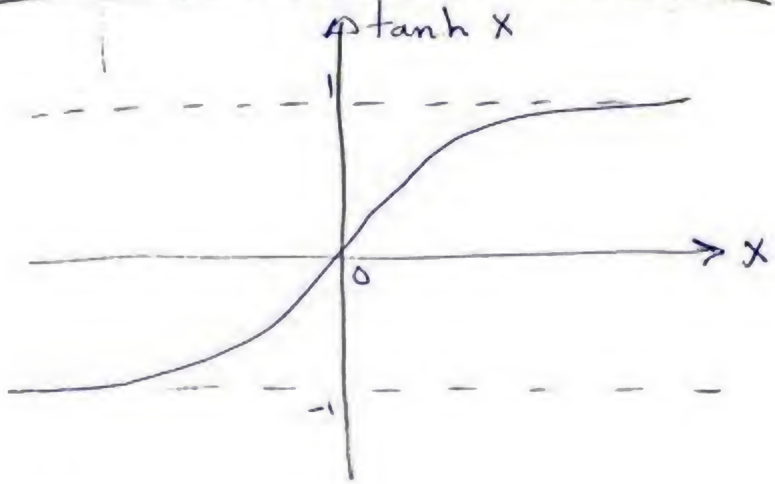


$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$



$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$= \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$



$$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} = \frac{1 - e^{-2x}}{1 + e^{-2x}}$$

$$= \frac{2}{1 + e^{-2x}} - 1$$

← المعادلة دى هى بالظبط معادلة ال [Bipolar sigmoidal function]

لكن عند $\boxed{\alpha = 2}$

Generalization, $\tanh x =$

$$\frac{2}{1+e^{-\alpha x}} - 1 = \frac{1-e^{-\alpha x}}{1+e^{-\alpha x}}$$

$$= \frac{e^{\frac{\alpha x}{2}} - e^{-\frac{\alpha x}{2}}}{e^{\frac{\alpha x}{2}} + e^{-\frac{\alpha x}{2}}} = \boxed{\tanh\left(\frac{\alpha x}{2}\right)}$$

لو وضعت $\alpha = 2$ هتطلعلك $(\tanh x)$